The n-wave procedure and dimensional regularization for the scalar field in a homogeneous isotropic space

Yu. V. Pavlov [*]

Institute of Mechanical Engineering, Russian Academy of Sciences,
61 Bolshoy, V.O., St. Petersburg, 199178, Russia;

A. Friedmann Laboratory for Theoretical Physics,
St. Petersburg, Russia

Abstract. We obtain expressions for the vacuum expectations of the energy-momentum tensor of the scalar field with an arbitrary coupling to the curvature in an N-dimensional homogeneous isotropic space for the vacuum determined by diagonalization of the Hamiltonian. We generalize the n-wave procedure to N-dimensional homogeneous isotropic space-time. Using the dimensional regularization, we investigate the geometric structure of the terms subtracted from the vacuum energy-momentum tensor in accordance with the n-wave procedure. We show that the geometric structures of the first three subtractions in the n-wave procedure and in the effective action method coincide. We show that all the subtractions in the n-wave procedure in a four- and five-dimensional homogeneous isotropic spaces correspond to a renormalization of the coupling constants of the bare gravitational Lagrangian.

Key words: scalar field, quantum field theory in curved space, renormalization, dimensional regularization.
PACS number: 04.62.+v

1. Introduction

Quantum field theory in a curved space-time is intensively developed since the 70s of the last century (see books [1], [2]) and has important applications to the cosmology and astrophysics. In particular, the particle creation

---

[*]E-mail: pavlov@lpt.ipme.ru

from the vacuum by a gravitational field can be used to explain the observed density of the visible and dark matter [3]. There exist models explaining the observed acceleration of the expansion of the Universe by effects of quantum field theory in a curved space [4].

In investigating the effect of a quantized field on the space-time metric, it is necessary to evaluate the vacuum expectations of the energy-momentum tensor (EMT), which are sources of the gravitational field in accordance with the semiclassical approach. Because the vacuum expectations of the EMT are represented by divergent expressions, renormalization is required. There exist different methods for eliminating divergences of the vacuum EMT (see the reviews in [1] and [2]). The n-wave procedure, proposed in [5], and the equivalent subtraction scheme [6], called the adiabatic regularization, are widely used for homogeneous isotropic spaces. They have been used in a great number of calculations (see [1], [2] for the scalar field with the conformal coupling to curvature and see, e.g., [7], [8] for an arbitrary coupling). Rigorously justifying the expressions obtained for the vacuum EMT requires interpreting the relevant subtraction schemes in terms of renormalized constants of the bare Lagrangian. For the scalar field with an arbitrary coupling to curvature, the geometric structure of infinite subtractions in the adiabatic procedure in a homogeneous isotropic space was established in [9]. But the geometric structure of finite counterterms and their interpretation in terms of renormalization have remained unexplored. For a space-time with flat spatial sections, the complete geometric structure of the counterterms in the n-wave procedure was determined using the dimensional regularization in [10], [11]. In [11], it was shown that all the subtractions in the n-wave procedure in the four-dimensional spase-time with flat spatial sections correspond to a renormalization of the constants of the bare dimensionally regularized gravitational Lagrangian.

This work is devoted to investigating the geometric structure of counterterms in the n-wave procedure for nonflat spatial sections and to comparing it with the results obtained from the dimensionally regularized effective action. We note that calculations for the N-dimensional space-time that are required for performing the dimensional regularization in this work can also be of independent interest for investigating quantum theory effects in curved space-time in higher-dimensional models.

In this work, we consider the complex scalar field with an arbitrary coupling to the curvature in the N-dimensional space-time with homogeneous isotropic spatial sections. In Sec. 2, we describe the geometric structure of the

expressions subtracted from the vacuun EMT in the effective action method. In Sec. 3, we obtain the expressions for the vacuum expectations of the EMT in the homogeneous isotropic space for the vacuum defined in accordance with the Hamiltonian diagonalization method. In Sec. 4, we generalize the n-wave procedure for the N-dimensional homogeneous isotropic case and give the first three counterterms, which exhaust all counterterms in dimensions $N = 4, 5$. Using dimensional regularization, we investigate the geometric structure of the n-wave procedure counterterms. In Sec. 5, the main results of the work are formulated. In Appendix A, we give some formulas for eigenfunctions of the Laplace-Beltrami operator in the $(N-1)$-dimensional space of constant curvature used to calculate the vacuum expectations of the EMT. In Appendix B, we give expressions for the geometric quantities in the N-dimensional homogeneous isotropic space-time encountered in this work.

We use the system of units where $\hbar = c = 1$. The signs of the curvature tensor and the Ricci tensor are chosen such that $R^{i}_{\ jkl} = \partial_l \Gamma^{i}_{\ jk} - \partial_k \Gamma^{i}_{\ jl} + \Gamma^{i}_{\ nl}\Gamma^{n}_{\ jk} - \Gamma^{i}_{\ nk}\Gamma^{n}_{\ jl}$ and $R_{ik} = R^{l}_{\ ilk}$, where $\Gamma^{i}_{\ jk}$ are Christoffel symbols.

2. Geometric structure of counterterms in the effective action method

We consider a complex scalar field $\varphi(x)$ of mass m with the equation of motion

$$(\nabla^i \nabla_i + \xi R + m^2)\varphi(x) = 0, \tag{1}$$

that corresponds to the Lagrangian

$$L(x) = \sqrt{|g|}\,[\,g^{ik}\partial_i\varphi^*\partial_k\varphi - (m^2 + \xi R)\,\varphi^*\varphi\,], \tag{2}$$

where ∇_i are covariant derivatives corresponding to the metric g_{ik}, $g = \det(g_{ik})$, R is the scalar curvature, and ξ is the coupling constant to the curvature. In the space-time of dimension N with $\xi = \xi_c \equiv (N-2)/[\,4\,(N-1)]$ (the conformal coupling) and $m = 0$, Eq. (1) is conformally invariant ($\xi_c = 1/6$ for $N = 4$).

The metric EMT, obtained by varying the action with respect to the metric, is given by (see [12])

$$T_{ik} = \partial_i\varphi^*\partial_k\varphi + \partial_k\varphi^*\partial_i\varphi - g_{ik}|g|^{-1/2}L(x) - 2\xi(R_{ik} + \nabla_i\nabla_k - g_{ik}\nabla^j\nabla_j)\varphi^*\varphi. \tag{3}$$

Expressions for the vacuum expectations of the EMT are devergent. In a space-time with a metric of the general form, it is convenient to analyze the geometric structure of the divergences using the dimensionally regularised effective action. For the complex scalar field $\varphi(x)$ with the equation of motion (1), the one-loop effective action can be written as (see [2], [13])

$$S_{eff} = \int L_{eff}(x)\sqrt{|g|}\,d^N x\,, \tag{4}$$

where

$$L_{eff}(x) = (4\pi)^{-N/2}\left(\frac{M}{m}\right)^{2\varepsilon}\sum_{j=0}^{\infty} a_j(x)\,m^{N_0-2j}\,\Gamma\left(j-\frac{N}{2}\right), \tag{5}$$

$$a_0(x) = 1\,, \qquad a_1(x) = \left(\frac{1}{6}-\xi\right)R\,, \tag{6}$$

$$a_2(x) = \frac{1}{180}R_{lmpq}R^{lmpq} - \frac{1}{180}R_{lm}R^{lm} - \frac{1}{6}\left(\frac{1}{5}-\xi\right)\nabla^l\nabla_l R + \frac{1}{2}\left(\frac{1}{6}-\xi\right)^2 R^2, \tag{7}$$

N is the space-time dimension, which is considered as variable analytically continued into the complex plane, ε is a complex parameter, M is a constant with the dimension of mass [14] introduced to preserve the standard dimension $(\text{length})^{-N_0}$ of L_{eff} in the case $N = N_0 - 2\varepsilon$, and $\Gamma(z)$ is the Gamma function.

The first $[N_0/2]+1$ terms in (5) are to be eliminated to obtain the renormalized Lagrangian L_{eff} ($[b]$ denotes the integer part of b). Varying the effective action terms corresponding to $j = 0, 1$ with respect to g_{ik}, we obtain the terms subtracted from the vacuum EMT,

$$T_{ik,\varepsilon}[0] = -\frac{m^{N_0}}{2^{N_0}\pi^{N_0/2}}\left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon}\Gamma\left(\varepsilon-\frac{N_0}{2}\right)g_{ik}\,, \tag{8}$$

$$T_{ik,\varepsilon}[1] = \frac{m^{N_0-2}}{2^{N_0-1}\pi^{N_0/2}}\left(\frac{1}{6}-\xi\right)\left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon}\Gamma\left(1-\frac{N}{2}\right)G_{ik} =$$

$$= \frac{m^{N_0-2}}{2^{N_0-1}\pi^{N_0/2}}\left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon}\left[\frac{-\Gamma\left(3-\frac{N}{2}\right)}{3(N-1)(N-2)} + \Delta\xi\,\Gamma\left(1-\frac{N}{2}\right)\right]G_{ik}\,, \tag{9}$$

where $G_{ik} = R_{ik} - Rg_{ik}/2$ is the Einstein tensor and $\Delta\xi \equiv \xi_c - \xi$.

We find the term $T_{ik,\varepsilon}[2]$, corresponding to subtraction of the term with $j = 2$ from L_{eff}. To analysis the case of homogeneous isotropic spaces, it is convenient to rewrite (7) as

$$a_2(x) = \frac{N-6}{720(N-3)}(R_{lmpq}R^{lmpq} - 4R_{lm}R^{lm} + R^2) + \frac{(N-2)C_{lmpq}C^{lmpq}}{240(N-3)} -$$

$$- \frac{1}{6}\left(\frac{1}{5} - \xi\right)\nabla^l\nabla_l R + \left(\frac{(N-4)(N-6)}{480(N-1)^2} + \frac{\Delta\xi(4-N)}{12(N-1)} + \frac{(\Delta\xi)^2}{2}\right)R^2, \quad (10)$$

where

$$C_{iklm} = R_{iklm} + \frac{2}{N-2}\left(R_{m[i}g_{k]l} - R_{l[i}g_{k]m}\right) + \frac{2R\,g_{l[i}\,g_{k]m}}{(N-1)(N-2)} \quad (11)$$

is the conformal Weyl tensor and the square brackets in subscripts denote antisymmetrization: $A_{n[i}B_{k]m} = (A_{ni}B_{km} - A_{nk}B_{im})/2$. Using (10) and varying the effective action term corresponding to $j = 2$ with respect to g_{ik}, we obtain $T_{ik,\varepsilon}[2]$ in the form

$$T_{ik,\varepsilon}[2] = \frac{m^{N_0-4}}{(4\pi)^{\frac{N_0}{2}}}\left(\frac{4\pi M^2}{m^2}\right)^\varepsilon\left[\frac{(N-6)E_{ik}}{360(N-3)}\Gamma\left(2-\frac{N}{2}\right) + \frac{(N-2)W_{ik}}{120(N-3)}\Gamma\left(2-\frac{N}{2}\right) +$$

$$+ \left(\frac{\Gamma\left(4-\frac{N}{2}\right)}{60(N-1)^2} + \Delta\xi\frac{\Gamma\left(3-\frac{N}{2}\right)}{3(N-1)} + (\Delta\xi)^2\Gamma\left(2-\frac{N}{2}\right)\right)^{(1)}H_{ik}\right], \quad (12)$$

where

$$^{(1)}H_{ik} = \frac{\delta\int R^2\sqrt{|g|}\,d^Nx}{\sqrt{|g|}\,\delta g^{ik}} = 2\left(\nabla_i\nabla_k R - g_{ik}\nabla^l\nabla_l R\right) + 2R\left(R_{ik} - \frac{1}{4}R\,g_{ik}\right), \quad (13)$$

$$E_{ik} = \frac{1}{\sqrt{|g|}}\frac{\delta}{\delta g^{ik}}\int(R_{lmpq}R^{lmpq} - 4R_{lm}R^{lm} + R^2)\sqrt{|g|}\,d^Nx, \quad (14)$$

$$W_{ik} = \frac{1}{\sqrt{|g|}}\frac{\delta}{\delta g^{ik}}\int C_{lmpq}C^{lmpq}\sqrt{|g|}\,d^Nx. \quad (15)$$

In the four-dimensional space-time, $E_{ik} = 0$ because

$$\frac{\delta}{\delta g^{ik}}\int\left(R^{lmpq}R_{lmpq} - 4R^{lm}R_{lm} + R^2\right)\sqrt{|g|}\,d^4x = 0 \quad (16)$$

in accordance with the Gauss-Bonnet theorem. Using (11), (13) and the formulas

$$^{(2)}H_{ik} = \frac{1}{\sqrt{|g|}} \frac{\delta}{\delta g^{ik}} \int R^{lm} R_{lm} \sqrt{|g|}\, d^N x = \nabla_i \nabla_k R - \nabla^l \nabla_l R_{ik} -$$

$$- \frac{1}{2} \left(\nabla^l \nabla_l R + R^{lm} R_{lm} \right) g_{ik} + 2 R^{lm} R_{limk}\,, \tag{17}$$

$$H_{ik} = \frac{1}{\sqrt{|g|}} \frac{\delta}{\delta g^{ik}} \int R^{lmpq} R_{lmpq} \sqrt{|g|}\, d^N x = 2\nabla_i \nabla_k R - 4 \nabla^l \nabla_l\, R_{ik} -$$

$$- 4 R_{il} R_k^l + 4 R^{lm} R_{limk} - \frac{1}{2} g_{ik} R_{lmpq} R^{lmpq} + 2 R_{ilmp} R_k^{\ lmp} \tag{18}$$

for variational derivatives of the expressions that are quadratic in the curvature (see [15]), we can obtain (see [13])

$$E_{ik} = H_{ik} - 4\,^{(2)}H_{ik} + \,^{(1)}H_{ik} = 2 C_{ilmp} C_k^{\ lmp} - \frac{g_{ik}}{2} C_{lmpq} C^{lmpq} - (N-4)\,^{(3)}H_{ik} \tag{19}$$

for arbitrary N, where

$$^{(3)}H_{ik} = \frac{4}{N-2} C_{ilkm} R^{lm} + \frac{2(N-3)}{(N-2)^2} \left[2 R_{il} R_k^l - \frac{N}{N-1} R R_{ik} - \right.$$

$$\left. - g_{ik} \left(R_{lm} R^{lm} - \frac{N+2}{4(N-1)} R^2 \right) \right]. \tag{20}$$

In the conformally flat case ($C_{iklm} = 0$), the tensor $^{(3)}H_{ik}$ is covariantly conserved, which follows from (19) for $N \neq 4$ and can be directly verified for $N = 4$. The tensor $^{(3)}H_{ik}$ proposed in [13] generalizes the corresponding tensor introduced in [16] for the conformally flat four-dimensional case.

As $N \to 4$, the product $E_{ik}\Gamma(2 - (N/2))$ has a finite limit for an arbitrary space-time metric because in the analitic continuation with respect to the dimension, the dependence of the expressions on N is assumed to be given by a rational function, $E_{ik} = 0$ at $N = 4$, and the Gamma function has a first-order pole at zero. Therefore, the first term in the square brackets in the right-hand side of (12) is always finite as $N \to 4$. If this term is not subtracted in renormalization the EMT, then the vacuum EMT is finite as $N \to 4$, but the effective action then remains divergent, and the expression for the anomalous trace of the vacuum EMT is differnt from the standard

one. Under an additional finite renormalization of the coefficient at R^2 in L_{eff} (see a discussion of this point in Sec. 6.3 in [2]), the anomalous trace can be made vanishing for the conformal scalar field in the conformally flat space. This occurs, e.g., in [17], where the time-dependent normal ordering of operators was used to obtain finite values of the vacuum EMT in a homogeneous isotropic space for the conformal scalar field.

In what follows, in accordance with standard approach, we keep the term with E_{ik} in (12). In the conformally flat case, in particular, in a homogeneous isotropic space-time, taking the equalities $C_{iklm} = 0, W_{ik} = 0$ and (19) into account, we obtain the following expression for the term $T_{ik,\varepsilon}[2]$ that is subtracted from the vacuum EMT in the effective action method:

$$
T_{ik,\varepsilon}[2] = \frac{m^{N_0-4}}{(4\pi)^{N_0/2}} \left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon} \left[\frac{-^{(3)}H_{ik}}{90(N-3)} \Gamma\left(4 - \frac{N}{2}\right) + \right.
$$

$$
\left. + \left(\frac{\Gamma\left(4 - \frac{N}{2}\right)}{60\,(N-1)^2} + \Delta\xi\,\frac{\Gamma\left(3 - \frac{N}{2}\right)}{3\,(N-1)} + (\Delta\xi)^2\,\Gamma\left(2 - \frac{N}{2}\right) \right)^{(1)}H_{ik} \right]. \tag{21}
$$

3. Scalar field in the homogeneous isotropic space

The metric of the N-dimensional homogeneous isotropic space can be written as

$$
ds^2 = g_{ik}\,dx^i dx^k = a^2(\eta)\left(d\eta^2 - dl^2\right), \tag{22}
$$

where $dl^2 = \gamma_{\alpha\beta}\,dx^\alpha dx^\beta$ is the metric of the $(N-1)$-dimensional space of constant curvature $K = 0, \pm 1$. The complete set of solutions of Eq. (1) in metric (22) can be found in the form

$$
\varphi(x) = a^{-(N-2)/2}(\eta)\,g_\lambda(\eta)\Phi_J(\mathbf{x}), \tag{23}
$$

where

$$
g_\lambda''(\eta) + \Omega^2(\eta)\,g_\lambda(\eta) = 0, \tag{24}
$$

$$
\Omega^2(\eta) = m^2 a^2 + \lambda^2 - \Delta\xi\,a^2 R, \tag{25}
$$

$$
\Delta_{N-1}\,\Phi_J(\mathbf{x}) = -\left(\lambda^2 - \left(\frac{N-2}{2}\right)^2 K\right)\Phi_J(\mathbf{x}), \tag{26}
$$

the prime denotes the derivative with respect to the conformal time η, and J is a set of indices (quantum numbers) that label the eigenfunctions of

the Laplace-Beltrami operator Δ_{N-1} in the $(N-1)$-dimensional space. The nonnegativity of the eigenvalues of the operator $-\Delta_{N-1}$ implies the inequality $\lambda^2 - ((N-2)/2)^2\, K \geq 0$.

In according with the Hamiltonian diagonalization method [1] (see [18] for the case of an arbitrary ξ), the function $g_\lambda(\eta)$ must satisfy the initial conditions

$$g'_\lambda(\eta_0) = i\,\Omega(\eta_0)\, g_\lambda(\eta_0)\,, \quad |g_\lambda(\eta_0)| = \frac{1}{\sqrt{\Omega(\eta_0)}}\,. \tag{27}$$

For quantizing, we expand the field $\varphi(x)$ with respect to the complete set of solutions of Eq. (23)

$$\varphi(x) = \int d\mu(J) \left[\varphi^{(+)}_J\, a^{(+)}_J + \varphi^{(-)}_J\, a^{(-)}_J \right]\,, \tag{28}$$

where

$$\varphi^{(+)}_J(x) = \frac{a^{-(N-2)/2}(\eta)}{\sqrt{2}}\, g_\lambda(\eta)\, \Phi^*_J(\mathbf{x})\,, \quad \varphi^{(-)}_J(x) = \left(\varphi^{(+)}_J(x) \right)^*\,, \tag{29}$$

and impose the commutation relations

$$\left[a^{(-)}_J,\ \overset{*}{a}{}^{(+)}_{J'} \right] = \left[\overset{*}{a}{}^{(-)}_J,\ a^{(+)}_{J'} \right] = \delta_{JJ'}\,, \quad \left[a^{(\pm)}_J,\ a^{(\pm)}_{J'} \right] = \left[\overset{*}{a}{}^{(\pm)}_J,\ \overset{*}{a}{}^{(\pm)}_{J'} \right] = 0\,. \tag{30}$$

It is convenient to express the EMT expectation values for the vacuum $|0\rangle$ annihilated by the operators $a^{(-)}_J$, $\overset{*}{a}{}^{(-)}_J$ in terms of the bilinear combinations of the functions g_λ and g^*_λ,

$$S = \frac{|g'_\lambda|^2 + \Omega^2\,|g_\lambda|^2}{4\,\Omega} - \frac{1}{2}\,, \quad U = \frac{\Omega^2\,|g_\lambda|^2 - |g'_\lambda|^2}{2\,\Omega}\,, \quad V = -\frac{d(g^*_\lambda g_\lambda)}{2\,d\eta}\,, \tag{31}$$

which in accordance with (24) satisfy the system of differential equations

$$S' = \frac{\Omega'}{2\,\Omega}\,U\,, \quad U' = \frac{\Omega'}{\Omega}\,(1 + 2S) - 2\,\Omega V\,, \quad V' = 2\,\Omega\,U\,. \tag{32}$$

Recalling the initial conditions $S(\eta_0) = U(\eta_0) = V(\eta_0) = 0$ following from (27), the Eqs. (32) can be rewritten as a system of the Volterra integral equations

$$U(\eta) + iV(\eta) = \int_{\eta_0}^{\eta} w(\eta_1)\,(1 + 2S(\eta_1))\,\exp[2\,i\,\Theta(\eta_1, \eta)]\,d\eta_1\,, \tag{33}$$

$$S(\eta) = \frac{1}{2} \int_{\eta_0}^{\eta} d\eta_1 \, w(\eta_1) \int_{\eta_0}^{\eta_1} w(\eta_2) \, (1 + 2S(\eta_2)) \cos[2\,\Theta(\eta_2, \eta_1)] \, d\eta_2 \,, \qquad (34)$$

where

$$w(\eta) = \frac{\Omega'(\eta)}{\Omega(\eta)} \,, \qquad \Theta(\eta_1, \eta_2) = \int_{\eta_1}^{\eta_2} \Omega(\eta) \, d\eta \,.$$

Substituting expression (28) in (3) and using (30), (31) and formulas (A.1)–(A.4) (see Appendix A), we obtain the (divergent) expressions for the vacuum expectations of the EMT,

$$\langle 0 | \, T_{ik} \, | 0 \rangle = \frac{B_N}{a^{N-2}} \int \tau_{ik} \, d\mu(\lambda) \,, \qquad (35)$$

where $B_N = \left[2^{N-3} \pi^{(N-1)/2} \, \Gamma((N-1)/2) \right]^{-1}$ and "spectral densities" τ_{ik} are given by

$$\tau_{00} = \Omega \left(S + \frac{1}{2} \right) + \Delta\xi \, (N{-}1) \left[cV + \left(c' + (N{-}2)c^2 \right) \frac{1}{\Omega} \left(S + \frac{1}{2}U + \frac{1}{2} \right) \right], \quad (36)$$

$$\tau_{\alpha\beta} = \gamma_{\alpha\beta} \left\{ \frac{\lambda^2}{(N-1)\,\Omega} \left(S + \frac{1}{2} \right) - \frac{\Omega^2 - \lambda^2}{2\,(N-1)\,\Omega} U - \right. \qquad (37)$$

$$\left. - \Delta\xi \left[\left((N{-}1)\,c' + (N{-}2)\,K \right) \frac{1}{\Omega} \left(S + \frac{1}{2}U + \frac{1}{2} \right) + 2\,\Omega\,U - (N{-}1)\,c\,V \right] \right\},$$

and $c \equiv a'/a$. The integration measure in (35) in the four-dimensional space-time is (see [1])

$$\int d\mu(\lambda) \ldots = \begin{cases} \displaystyle\int_0^\infty d\lambda \, \lambda^2 \ldots, & K = 0, -1 \,, \\[2mm] \displaystyle\sum_{\lambda=1}^\infty \lambda^2 \ldots, & K = 1. \end{cases} \qquad (38)$$

In the N-dimensional case,

$$\int d\mu(\lambda) \ldots = \int d\lambda \, f(\lambda) 2 B_N^{-1} \ldots \,,$$

where the function $f(\lambda)$ is defined in (A.1) and can be written in the respective forms (A.6) and (A.8) in quasi-Euclidean ($K = 0$) and spherical ($K = 1$) cases.

We note that the expressions for the vacuum expectation of the EMT given in [7] are valid only for $N = 4$, whereas they are evaluated in [1] for the vacuum state that does not correspond to the Hamiltonian diagonalization method (for $\xi \neq \xi_c$ and $R \neq 0$).

4. The n-wave procedure

The n-wave prosedure proposed in [5] is often used to calculate the renormalized vacuum EMT in homogeneous isotropic spaces. In the quasi-Euclidean $(K = 0)$ case, the procedure is as follows. If the vacuum expectations of some operator A bilinear in the field are expressed as

$$\langle 0|A|0\rangle = \int a(\lambda, m)\, d\lambda\,, \tag{39}$$

the corresponding renormalized value is then obtained by regularizing the contribution of each mode (i.e., of the integrand with a given λ). For this, the replacements $\lambda \to n\lambda$ and $m \to nm$ are made, and the terms of the $1/n$-asymptotic expansion $(n \to \infty)$ that make integral (39) divergent are subtracted from $a(\lambda, m)$. This procedure corresponds to subtraction of the contribution of the so-called n-waves (see [5] and Sec. 2.2 in [1] for more details). This method for eliminating divergences is equivalent to the adiabatic regularization method based on introducing the adiabaticity parameter for the metric variation and on subtracting the first terms of the asymptotic expansion of $a(\lambda, m)$ in inverse powers of this parameter in (39) (see [2]).

In applying the n-wave procedure in the case of a spherical $(K = 1)$ and a hyperbolic $(K = -1)$ N-dimensional space-time, it is necessery to take into account the difference of the measure in (35) from the quasi-Euclidean case. Writing the integration measure as $d\mu(\lambda) = \sigma(\lambda)\, d\lambda$, we take the generalization of the n-wave procedure to the N-dimensional homogeneous isotropic space-time to be given by the expression

$$\langle 0|\, T_{ik}|0\rangle_{ren} = \frac{B_N}{a^{N-2}} \lim_{\Lambda \to \infty} \left[\int^{\Lambda} \tau_{ik}\, d\mu(\lambda) - \sum_{l=0}^{[N/2]} \int_0^{\Lambda} \lambda^{N-2} a_{ik}[l]\, d\lambda \right], \tag{40}$$

where

$$a_{ik}[l] = \frac{1}{l!} \lim_{n \to \infty} \frac{\partial^l}{\partial (n^{-2})^l} \left(\frac{\tau_{ik}(n\lambda, nm)\, \sigma(n\lambda)}{n^{N-1}\lambda^{N-2}} \right). \tag{41}$$

In the four-dimensional case, this generalization coincides with the standard definition of the n-wave procedure. The function $\sigma(\lambda)$ accounts for the difference of the integration measure from $\lambda^{N-2}\, d\lambda$ for $K \neq 0$ and $N \neq 4$.

We now find the explicit form of $a_{ik}[l]$. For this, we expand S, U, and V (see (31)) in inverse powers of n and replace $\lambda \to n\lambda$ and $m \to nm$ with $n \to \infty$:

$$S = \sum_{k=1}^{\infty} n^{-k} S_k\,, \quad \ldots$$

Using consecutive iterations in integral equations (33), (34) and the stationary phase method, we obtain the first nonzero terms of the expansions,

$$V_1 = W, \quad U_2 = DW, \quad S_2 = \frac{1}{4}W^2, \quad V_3 = \frac{1}{2}W^3 - D^2W - \frac{\omega}{2}D\left(\frac{q}{\omega^3}\right), \quad (42)$$

$$U_4 = \frac{3}{2}W^2DW - D^3W - D\left(\frac{\omega}{2}D\left(\frac{q}{\omega^3}\right)\right) + \frac{q}{2\omega^2}DW, \quad (43)$$

$$S_4 = \frac{3}{16}W^4 + \frac{1}{4}(DW)^2 - \frac{1}{2}WD^2W - \frac{1}{4}\omega WD\left(\frac{q}{\omega^3}\right), \quad (44)$$

where

$$q = \Delta\xi\, a^2 R, \quad \omega = (m^2 a^2 + \lambda^2)^{1/2}, \quad W = \frac{\omega'}{2\,\omega^2}, \quad D = \frac{1}{2\,\omega}\frac{d}{d\eta}. \quad (45)$$

It must be note that in calculating the quantities S_k, U_k, and V_k, as in [1], [5], [10], we neglect the terms depending on the initial time instant η_0, i.e., restrict ourself to the case of counterterms that are local in the time η. In particular, nonlocal terms are absent whenever it is assumed that the first $2[N/2]$ derivatives of the scale factor $a(\eta)$ of the metric vanish at the initial time instant.

We use (36), (37), (42)–(44) and in what follows set the measure $\sigma(\lambda)\,d\lambda$ equal to

$$\sigma(\lambda) = \lambda^{N-2} + \alpha_N\lambda^{N-4} + \beta_N\lambda^{N-6} + \dots, \quad (46)$$

where $\alpha_N, \beta_N, \dots$ are the functions of N and K, which vanish at $N = 4$, to be determined below. As a result, we obtain the expressions for $a_{ik}[l]$

$$a_{00}[0] = \tau_{00}[0] = \frac{\omega}{2}, \quad a_{\alpha\beta}[0] = \tau_{\alpha\beta}[0] = \gamma_{\alpha\beta}\frac{\lambda^2}{2(N-1)\,\omega}, \quad (47)$$

$$a_{ik}[1] = \tau_{ik}[1] + \frac{\alpha_N}{\lambda^2}\tau_{ik}[0], \quad a_{ik}[2] = \tau_{ik}[2] + \frac{\alpha_N}{\lambda^2}\tau_{ik}[1] + \frac{\beta_N}{\lambda^4}\tau_{ik}[0], \quad (48)$$

where

$$\tau_{00}[1] = \omega\, S_2 + \Delta\xi\,(N-1)\left[c\,V_1 + \frac{N-2}{4\,\omega}(c^2 - K)\right], \quad (49)$$

$$\tau_{\alpha\beta}[1] = \frac{\gamma_{\alpha\beta}}{N-1}\left[\frac{\lambda^2}{\omega}S_2 - \frac{m^2a^2}{2\,\omega}\left(U_2 + \frac{q}{2\,\omega^2}\right)\right] + \gamma_{\alpha\beta}\Delta\xi\left[(N-1)\,c\,V_1 - \right.$$

$$\left. - 2\omega\left(U_2 + \frac{q}{2\,\omega^2}\right) + \frac{1}{4\,\omega}\left(-4\xi a^2 R + (N-2)\left(c^2(N-1) + K(N-3)\right)\right)\right], \quad (50)$$

$$\tau_{00}[2] = \omega \left(S_4 + \frac{q}{4\,\omega^2}\,U_2 + \frac{q^2}{16\,\omega^4} \right) + \tag{51}$$

$$+\Delta\xi(N-1)\left[c\,V_3 + \frac{N-2}{2\,\omega}\,(c^2 - K)\left(S_2 + \frac{1}{2}\,U_2 + \frac{q}{4\,\omega^2} \right) \right],$$

$$\tau_{\alpha\beta}[2] = \gamma_{\alpha\beta}\left\{ \frac{1}{N-1}\left[\frac{\lambda^2}{\omega}\left(S_4 + \frac{q}{4\,\omega^2}\,U_2 + \frac{q^2}{16\,\omega^4} \right) - \frac{m^2 a^2}{2\,\omega}\left(U_4 + \frac{q^2}{4\,\omega^4} + \right. \right.$$

$$\left. + \frac{q}{\omega^2}\,S_2 \right)\right] + \Delta\xi\left[(N-1)\,c\,V_3 - 2\,\omega\left(U_4 + \frac{q^2}{4\,\omega^4} + \frac{q}{\omega^2}\,S_2 \right) + \tag{52}$$

$$+ \frac{1}{2\omega}\left(-4\xi a^2 R + (N-2)\left(c^2(N-1) + K(N-3) \right)\right)\left(S_2 + \frac{1}{2}U_2 + \frac{q}{4\omega^2} \right)\right]\right\}.$$

These expressions exhaust all the subtractions in dimensions $N = 4, 5$. Additional counterterms occur for $N \geq 6$ (they are given in [11] for the conformal scalar field with $K = 0$).

The vacuum EMT renormalized in accordance with (40) is covariantly conserved. This follows from Eqs. (47), (48) and the equalities $\nabla^i(\tau_{ik}/a^{N-2}) = 0$ and $\nabla^i(\tau_{ik}[l]/a^{N-2}) = 0$, which can be verified using (32), (36), (37), (49)–(52).

To find the geometric structure of the n-wave procedure counterterms, we use dimensional regularization, setting, as in Sec. 2, $N = N_0 - 2\varepsilon$, where ε is a complex parameter. In calculating integrals in the dimensionally regularized counterterms

$$T_{ik,\varepsilon}[l] = \frac{B_N}{a^{N-2}}(M)^{2\varepsilon}\int_0^\infty \lambda^{N-2} a_{ik,\varepsilon}[l]\,d\lambda, \tag{53}$$

where $a_{ik,\varepsilon}[l]$ are defined as in (47)–(52) with replacement $N \to N_0 - 2\varepsilon$, we use the equality

$$\int_0^\infty x^k\,(1 + x^2)^{-p}\,dx = \frac{\Gamma\left(\frac{k+1}{2}\right)\Gamma\left(p - \frac{k+1}{2}\right)}{2\,\Gamma(p)} \tag{54}$$

and the analitic continuation of the expression in its right-hand side. The result for the zeroth counterterm coincides with (8). Thus, both in the effective action method and in the n-wave procedure, the zeroth subtraction corresponds to renormalization of the cosmological constant.

With formulas (B.4), (B.9)–(B.12) (see Appendix B) taken into account, the structure of the first and the second subtractions in the n-wave procedure

in a homogeneous isotropic space-time can be obtained by the respective additions of $\Delta T_{ik,\varepsilon}[1]$ and $\Delta T_{ik,\varepsilon}[2]$ to (9) and (21), where

$$\Delta T_{ik,\varepsilon}[1] = \frac{m^{N_0-2}}{2^{N_0-1}\pi^{N_0/2}}\left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon} D_{ik}[1]\,, \tag{55}$$

$$\Delta T_{ik,\varepsilon}[2] = \frac{m^{N_0-4}}{(4\pi)^{N_0/2}}\left(\frac{4\pi M^2}{m^2}\right)^{\varepsilon} D_{ik}[2]\,, \tag{56}$$

$$D_{00}[1] = -\frac{1}{6}\Gamma\left(3-\frac{N}{2}\right)\left(K+\frac{24\,\alpha_N}{(N-2)(N-3)(N-4)}\right), \tag{57}$$

$$D_{\alpha\beta}[1] = \gamma_{\alpha\beta}\frac{N-3}{6(N-1)}\Gamma\left(3-\frac{N}{2}\right)\left(K+\frac{24\,\alpha_N}{(N-2)(N-3)(N-4)}\right), \tag{58}$$

$$D_{00}[2] = \frac{N-2}{a^2\,36}\left\{\left[\Gamma\left(4-\frac{N}{2}\right)(N-4)c^2 + \Delta\xi\,\Gamma\left(3-\frac{N}{2}\right)6(N-1)\,\times\right.\right.$$
$$\times\left.\left((N-2)K+(N-6)c^2\right)\right]\left[K+\frac{24\,\alpha_N}{(N-2)(N-3)(N-4)}\right]+$$
$$+\,\Gamma\left(4-\frac{N}{2}\right)\left(\frac{(5N-8)K^2}{10}-\frac{576\,\beta_N}{(N-2)(N-3)(N-4)(N-5)(N-6)}\right)\right\}, \tag{59}$$

$$D_{\alpha\beta}[2] = \frac{\gamma_{\alpha\beta}(N-2)}{a^2\,36(N-1)}\left\{\left[\Gamma\left(4-\frac{N}{2}\right)(N-4)\left((5-N)c^2-2c'\right)+\right.\right.$$
$$+\,\Delta\xi\Gamma\left(3-\frac{N}{2}\right)6(N-1)\left((5-N)(N-2)K+(N-6)\left((5-N)c^2-2c'\right)\right)\right]\times$$
$$\times\left[K+\frac{24\,\alpha_N}{(N-2)(N-3)(N-4)}\right]+\Gamma\left(4-\frac{N}{2}\right)\frac{(5-N)(5N-8)}{10}\,\times$$
$$\times\left.\left(K^2-\frac{5760\,\beta_N}{(N-2)(N-3)(N-4)(N-5)(N-6)(5N-8)}\right)\right\}. \tag{60}$$

Therefore, for the geometric structure of the first and the second subtractions in the n-wave procedure and in the effective action method to coincide, it is necessery that the same equalities (A.10), (A.11) that are obtained in Appendix A for integer values of N and $K = 0, 1$ be satisfied for arbitrary N. For $N = 4$, we have $\alpha_N = \beta_N = 0$, which agrees with (38). Thus, there exists a continuation with respect to the dimension of the integration measure

in the momentum space $\{\lambda\}$ such that the geometric structures of all the subtractions in the n-wave procedure for the four-dimensional homogeneous isotropic space-time and in the effective action method coincide.

The evaluated counterterms have a purely geometric structure (see (8), (9), and (21)) and can be represented as variational derivatives of geometric quantities with respect to the metric. Therefore, a Lagrangian for the gravitational field can be constructed such that adding these counterterms leads to a renormalization of its parameters (see a discussion of this approach in Sec. 6.11 in [19] and also in Sec. 6.1 in [2]). Because the counterterms contain expressions that are quadratic in the curvature, such a bare gravitational Lagrangian obviously does not coincide with the standard Einstein Lagrangian. The existence of expressions quadratic in the curvature indicates that quantization of fields in curved space inevitably involves going beyond the Einstein theory of gravity (Sec. 8.4 in [1]) The values of the renormalized constants must be determined experimentally, and as indicated, e.g., in [1], it is possible that the renormalized constants at the quadratic terms are equal to zero.

Using formulas (8), (9), (21), and

$$
R^{lmpq}R_{lmpq} - 4R^{lm}R_{lm} + R^2 = C_{lmpq}C^{lmpq} - \frac{4(N-3)}{N-2}\left(R_{lm}R^{lm} - \frac{N\,R^2}{4(N-1)}\right),
$$
(61)

we can draw the next conclusion. The first three n-wave procedure subtractions in the N-dimensional homogeneous isotropic space-time correspond to a renormalization of the cosmological and gravitational constants and of the parameters at the terms in the bare gravitational Lagrangian that are quadratic in the curvature and have the form

$$
L_{gr,\varepsilon} = \sqrt{|g|}\left[\frac{1}{16\pi G_\varepsilon}\left(R - 2\Lambda_\varepsilon\right) + \alpha_\varepsilon\left(R^{ik}R_{ik} - \frac{N\,R^2}{4(N-1)}\right) + \beta_\varepsilon\,R^2\right].
$$
(62)

For the conformal scalar field ($\xi = \xi_c$), the parameters G_ε and β_ε have a finite renormalization as $N \to 4$ (see (9) and (21)). But as $N \to 4$, the renormalization of the parameter α_ε in accordance with (12), (14), and (61) is infinite ($\sim (N-4)^{-1}$) for any value of the coupling constant ξ of the scalar field to the curvature.

5. Conclusions

We have investigated the geometric structure of counterterms of the vacuum EMT of a scalar field with an arbitrary coupling to the curvature in the N-dimensional homogeneous isotropic spase-time. The obtained formulas (35)–(37) determine nonrenormalized expectations of the EMT of a complex scalar field with an arbitrary coupling to the curvature, taken in the vacuum determined by the Hamiltonian diagonalization method [18]. The geometric structure of counterterms were determined in the effective action method for the N-dimensional homogeneous isotropic space (see (8), (9), and (21)). We have generalized the n-wave procedure to homogeneous isotropic N-dimensional spaces and determined the corresponding counterterms (47)–(52). We have found the properties (see (A.2)–(A.4)) of eigenfunctions of the Laplace-Beltrami operator in higher-dimensional homogeneous isotropic spaces, which are necessary for calculating the vacuum EMT. The geometric structure of counterterms in the n-wave procedure has been analyzed using dimensional regularization. We have found an analitic continuation with respect to the dimension of the integration measure in the momentum space (see (46), (A.10), and (A.11)). We have shown the coincidence of the geometric structures of the first three subtraction in the n-wave procedure and in the effective action method. We have shown that the first three subtractions (which exhaust all subtractions in dimensions $N = 4, 5$) of the n-wave procedure in the N-dimensional homogeneous isotropic space-time correspond to renormalization of the cosmological and gravitational constants and of the parameters at the terms in the bare gravitational Lagrangian that are quadratic in the curvature (62).

Comparison of the n-wave procedure subtractions (47)–(52) for $N = 4$ with the counterterms in the adiabatic regularization method, given in [9], demonstrates the equivalence of these methods for eliminating divergences. Therefore, conclusions regarding the geometric structure of subtractions obtained in this work for the n-wave procedure in a four-dimensional homogeneous isotropic space-time are also valid in the adiabatic regularization method.

Appendix A

We give some properties of the complete orthonormal set of the eigenfunctions $\Phi_J(\mathbf{x})$ of the Laplace-Beltrami operator Δ_{N-1} in an $(N-1)$-dimensional

space with the metric $dl^2 = \gamma_{\alpha\beta}\, dx^\alpha dx^\beta$ of constant curvature $K = 0, \pm 1$. These properties are used to calculate the vacuum expectations of the EMT. We set

$$f(\lambda) = \sum_{J\,(\lambda=\text{const})} |\Phi_J(\mathbf{x})|^2\,, \tag{A.1}$$

where λ is defined in (26) and where summation is replaced with integration for continuous J. In the case of a homogeneous isotropic space, the function $f(\lambda)$ is independent of $\mathbf{x}$. Applying the operator Δ_{N-1} to (A.1) and taking (26) into account, we obtain

$$\sum_{J\,(\lambda=\text{const})} \gamma^{\alpha\beta} \partial_\alpha \Phi_J^*(\mathbf{x})\, \partial_\beta \Phi_J(\mathbf{x}) = \left(\lambda^2 - \left(\frac{N-2}{2}\right)^2 K\right) f(\lambda)\,. \tag{A.2}$$

From the space isotropy condition with (A.2) taken into account, we obtain

$$\sum_{J\,(\lambda=\text{const})} \partial_\alpha \Phi_J^*(x)\, \partial_\beta \Phi_J(x) = \frac{\gamma_{\alpha\beta}}{N-1} \left(\lambda^2 - \left(\frac{N-2}{2}\right)^2 K\right) f(\lambda)\,. \tag{A.3}$$

Applying the covariant derivatives $\tilde\nabla_\alpha \tilde\nabla_\beta$ in the $(N-1)$-dimensional space to (A.1) and recalling (A.3), we obtain

$$\sum_{J\,(\lambda=\text{const})} \left[\left(\tilde\nabla_\alpha \tilde\nabla_\beta \Phi_J^*\right) \Phi_J + \Phi_J^* \tilde\nabla_\alpha \tilde\nabla_\beta \Phi_J\right] = -\frac{2\gamma_{\alpha\beta}}{N-1} \left(\lambda^2 - \left(\frac{N-2}{2}\right)^2 K\right) f(\lambda)\,. \tag{A.4}$$

In the quasi-Euclidean case ($K = 0$) in Cartesian coordinates,

$$\Phi_J(\mathbf{x}) = (2\pi)^{-(N-1)/2} \exp(-i\lambda_\alpha x^\alpha)\,, \tag{A.5}$$

where $-\infty < \lambda_\alpha < +\infty$. Integrating over the sphere of radious λ in the $(N-1)$-dimensional space of "dimensionless momenta" $\{\lambda_\alpha\}$ gives

$$f(\lambda) = \frac{B_N}{2} \lambda^{N-2}\,, \qquad K = 0\,, \tag{A.6}$$

where B_N is defined in Sec. 3.

In the case where $K = 1$ (spherical space) for a fixed λ, the set of the indices J is finite, and the volum of the $(N-1)$-dimensional space is given by $S_N(1)$, the surface area of the unit sphere in the N-dimensional space. Therefore,

$$f(\lambda) = \frac{1}{S_N(1)} \int \left(\sum_{J\,(\lambda=\text{const})} \Phi_J^*(\mathbf{x})\Phi_J(\mathbf{x})\right) \sqrt{\gamma}\, d^{N-1}\mathbf{x}\,,$$

where $\gamma = \det(\gamma_{\alpha\beta})$. Changing the order of the summation and integration operations and recalling that $\Phi_J(\mathbf{x})$ are orthonormalized, we obtain

$$f(\lambda) = \dim \lambda / S_N(1) , \qquad (A.7)$$

where $\dim \lambda$ is the multiplicity of the eigenvalue $\lambda^2 - ((N-2)/2)^2$ of the operator $-\Delta_{N-1}$. Using the relation expressing the multiplicity of the eigenvalues of the Laplace-Beltrami operator on the sphere through the dimension of the space of harmonic polynomials (see, e.g., [20]), we obtain the formula

$$f(\lambda) = \frac{B_N}{2} \frac{\Gamma(n+N-2)}{\Gamma(n+1)} \lambda , \quad n = 0, 1, \ldots , \quad \lambda = n + \frac{N-2}{2} . \qquad (A.8)$$

For an arbitrary N and $K = 0, 1$, expressions (A.6) and (A.8) can be written as

$$f(\lambda) = \frac{B_N}{2} \left(\lambda^{N-2} + \alpha_N \lambda^{N-4} + \beta_N \lambda^{N-6} + \ldots \right), \qquad (A.9)$$

where

$$\alpha_N = -\frac{1}{24} (N-2)(N-3)(N-4) K , \qquad (A.10)$$

$$\beta_N = \frac{1}{5760} (N-2)(N-3)(N-4)(N-5)(N-6)(5N-8) K^2 . \qquad (A.11)$$

Appendix B

We give expressions for some geometric quantities in the N-dimensional homogeneous isotropic space-time with metric (22).

The nonzero Christoffel symbols are

$$\Gamma^0_{00} = \frac{a'}{a} \equiv c , \quad \Gamma^\alpha_{0\beta} = c \, \delta^\alpha_\beta , \quad \Gamma^0_{\alpha\beta} = c \, \gamma_{\alpha\beta} , \quad \Gamma^\alpha_{\beta\delta}(g_{ik}) = \Gamma^\alpha_{\beta\delta}(\gamma_{\nu\mu}) . \quad (B.1)$$

The nonzero Ricci tensor components and the scalar curvature are

$$R_{00} = (N-1) \, c' , \quad R_{\alpha\beta} = -\gamma_{\alpha\beta} \left[c' + (N-2)(c^2 + K) \right], \qquad (B.2)$$

$$R = a^{-2}(N-1) \left[2c' + (N-2)(c^2 + K) \right]. \qquad (B.3)$$

The Einstein tensor components are

$$G_{00} = -\frac{(N-1)(N-2)}{2}(c^2 + K), \quad G_{\alpha\beta} = \gamma_{\alpha\beta}(N-2) \left[c' + \frac{(N-3)}{2}(c^2 + K) \right]. \qquad (B.4)$$

Using Eqs.(B.1)–(B.3), we obtain the square of the Ricci tensor and the second derivatives in the form:

$$R_{lm}R^{lm} = a^{-4}\,(N-1)\left[Nc'^{\,2} + 2(N-2)\,c'(c^2+K) + (N-2)^2(c^2+K)^2\right],$$
(B.5)

$$\nabla_0\nabla_0 R = a^{-2}\,2(N-1)\left[c^{(3)} + (N-7)\,c''c + (N-4)\,c'^{\,2} - \right.$$
$$\left. - 6(N-3)\,c'c^2 + 3(N-2)c^4 + K(N-2)(3c^2-c')\right],$$
(B.6)

$$\nabla_\alpha\nabla_\beta R = \gamma_{\alpha\beta}a^{-2}\,2(N-1)\left[-c''c - (N-4)c'c^2 + (N-2)c^2(c^2+K)\right],$$
(B.7)

$$\nabla^l\nabla_l R = a^{-4}\,2(N-1)\left[c^{(3)} + 2(N-4)\,c''c + (N-4)\,c'^{\,2} + \right.$$
$$+ (N^2-10N+20)\,c'c^2 - (N-2)((N-4)c^2+c')\,(c^2+K)\Big].$$
(B.8)

Using (B.2), (B.3), (B.5)–(B.8), we obtain the components $^{(1)}H_{ik}$ and $^{(3)}H_{ik}$ (see (13), (20)) as

$$^{(1)}H_{00} = \frac{(N-1)^2}{a^2}\left[2c'^{\,2} - 4c''c - 4(N-4)c'c^2 - \frac{c^4}{2}(N-2)(N-10) - \right.$$
$$\left. - K(N-2)\left(\frac{N-2}{2}K + (N-6)c^2\right)\right],$$
(B.9)

$$^{(1)}H_{\alpha\beta} = \gamma_{\alpha\beta}\,a^{-2}(N-1)\left\{4c^{(3)} + 4\,(2N-9)\,c''c + 2\,(3N-11)\,c'^{\,2} + \right.$$
$$+ 6\,(N^2-10N+20)\,c'c^2 + \frac{1}{2}(N-2)(N^2-15N+50)\,c^4 + $$
$$+ K(N-2)\left[(N-5)(N-6)c^2 + 2(N-6)c' + \frac{1}{2}(N-2)(N-5)K\right]\right\},$$
(B.10)

$$^{(3)}H_{00} = a^{-2}\,2^{-1}(N-1)(N-2)(N-3)\,(c^2+K)^2\,,$$
(B.11)

$$^{(3)}H_{\alpha\beta} = -\gamma_{\alpha\beta}\,a^{-2}\,2^{-1}(N-2)(N-3)\left[4c'(c^2+K) + (N-5)\,(c^2+K)^2\right].$$
(B.12)

References

[1] A. A. Grib, S. G. Mamayev and V. M. Mostepanenko, *Vacuum Quantum Effects in Strong Fields* [in Russian], Energoatomizdat, Moscow (1988) [English transl.: Friedmann Laboratory Publishing, St. Petersburg (1994)].

[2] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge (1982).

[3] A. A. Grib and Yu. V. Pavlov, *Superheavy particles in Friedmann cosmology and the dark matter problem*, Int. J. Mod. Phys. D **11**, 433 (2002); *Cold dark matter and primordial superheavy particles*, Int. J. Mod. Phys. A **17**, 4435 (2002).

[4] L. Parker and A. Raval, *Nonperturbative effects of vacuum energy on the recent expansion of the universe*, Phys. Rev. D **60**, 063512 (1999); *A new look at the accelerating universe*, Phys. Rev. Lett. **86**, 749 (2001).

[5] Ya. B. Zel'dovich and A. A. Starobinsky, *Particle creation and vacuum polarization in an anisotropic gravitational field*, ZhETF **61**, 2161 (1971) [English transl.: Sov. Phys.–JETP (USA) **34**, 1159 (1972)].

[6] L. Parker and S. A. Fulling, *Adiabatic regularization of the energy-momentum tensor of a quantized field in homogeneous spaces*, Phys. Rev. D **9**, 341 (1974).

[7] M. Bordag, J. Lindig and V. M. Mostepanenko, *Particle creation and vacuum polarization of a non-conformal scalar field near the isotropic cosmological singularity*, Class. Quantum Grav. **15**, 581 (1998).

[8] S. Habib, C. Molina-París and E. Mottola, *Energy-momentum tensor of particles created in an expanding universe*, Phys. Rev. D **61**, 024010 (2000).

[9] T. S. Bunch, *Adiabatic regularisation for scalar fields with arbitrary coupling to the scalar curvature*, J. Phys. A **13**, 1297 (1980).

[10] S. G. Mamayev, V. M. Mostepanenko and V. A. Shelyuto, *Dimensional regularization method for quantized fields in non-stationary isotropic*

spaces, Teor. Matem. Fiz. **63**, 64 (1985) [English transl.: Theor. Math. Phys. **63**, 366 (1985)].

[11] Yu. V. Pavlov, *Dimensional regularization and n-wave procedure for scalar fields in many-dimensional quasi-Euclidean spaces*, Teor. Matem. Fiz. **128**, 236 (2001) [English transl.: Theor. Math. Phys. **128**, 1034 (2001)].

[12] N. A. Chernikov and E. A. Tagirov, *Quantum theory of scalar field in de Sitter space-time*, Ann. Inst. H. Poincaré **A 9**, 109 (1968).

[13] T. S. Bunch, *On renormalization of the quantum stress tensor in curved space-time by dimensional regularization*, J. Phys. A **12**, 517 (1979).

[14] G. 't Hooft, *Dimensional regularization and the renormalization group*, Nucl. Phys. **B 61**, 455 (1973).

[15] B. S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach, New York (1965).

[16] V. L. Ginzburg, D. A. Kirzhnits and A. A. Lyubushin, *On the role of quantum fluctuations of a gravitational field in general relativity and cosmology*, ZhETF **60**, 451 (1971) [English transl.: Sov. Phys.–JETP (USA) **33**, 242 (1971)].

[17] S. G. Mamayev, V. M. Mostepanenko and A. A. Starobinsky, *Particle creation from the vacuum near a homogeneous isotropic singularity*, ZhETF **70**, 1577 (1976) [English transl.: Sov. Phys.–JETP (USA) **43**, 823 (1976)].

[18] Yu. V. Pavlov, *Nonconformal scalar field in a homogeneous isotropic space and the Hamiltonian diagonalization method*, Teor. Matem. Fiz. **126**, 115 (2001) [English transl.: Theor. Math. Phys. **126**, 92 (2001)].

[19] B. S. DeWitt, *Quantum fields in curved space-time*, Phys. Rept. C **19**, 295 (1975).

[20] M. A. Shubin, *Pseudodifferential Operators and Spectral Theory* [in Russian], Nauka, Moscow (1978) [English transl.: Springer, Berlin (2001)].